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ORIGINAL PAPER

A Production Inventory Model to Study the Supply Chain of Agri-Product for a Time Reliant Population

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Abstract
In the field of inventory management, the production inventory plays a vital role in optimizing the production rate and minimizing the cost involved in production and inventory control. In most of the cases it is observed that, demand is taken as function of price, time, stock, etc., however, the quantity of consumption by a particular population group is often neglected while developing a production inventory model. Further, to cope up with average consumption of a time dependent population in a certain region, it is necessary to determine the optimal production rate to maintain the supply chain. In this paper, an attempt has been made to develop a production inventory model to determine the optimum production rate of an agricultural product under the assumption of average consumption by a population. The objective function consists of setup cost, deterioration cost, transportation cost, carbon emission cost and the production cost. The model is solved under three different cases such as- considering holding cost, green investment, and preservation cost. Moreover, a comparison has been made among the different cases taken under study for least average total cost. Further, the model is validated with numerical examples followed by sensitivity analysis of the systems parameters.

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Introduction

Production inventory plays a vital role in real world mainly in business organization. Without production inventory, a company cannot take up production. Therefore, it is an essential part of supply chain process and directly affects each business' ability to meet the demand. Production inventory models are generally developed to ensure the optimum level of inventory that should be maintained during production to keep the ordering process of raw materials or goods. There are lots of factors like- production cost, holding cost, preservation cost etc., that involve while developing an inventory model. Market demand is one among such parameters which also largely affects while making an optimal ordering decision. In recent times, lots of models have been developed incorporating various types of demand pattern as like- price-dependent, stock-dependent, time-dependent demand etc. However, no research work has been reported in the literature of economic production quantity modelling where demand is a function of the average consumption rate of grain by population and the size of population in a certain region. The population size is function of time and follows the population decay model with migration and immigration factors. Thus, in this paper, for the first time, a production inventory model is developed where demand function represents the average consumption of a population and time-dependent population size.

The rest of this article is segmented as follows; Sect. 2 includes an elaborated literature review on production inventory and population growth models. Section 3 and Sect. 4 respectively give notations used and assumptions made in order to formulate the model. Section 5 contains description of the proposed model. The model is formulated in Sect. 6. Section 7 provides a solution procedure to find the optimal solution. To illustrate and validate the developed model, numerical example describing a hypothetical system is considered in Sect. 8. In Sect. 9, special cases regarding holding cost, green investment and preservation investment are provided together with numerical illustrations for each of the cases. The convexity of the average cost function with respect to the problem undertaken is verified graphically wherever is it is applicable. In Sect. 10, sensitivity analysis of system parameters is provided followed by some managerial insights in Sect. 11. Finally, in Sect. 12, conclusion is drawn and further possible extension of this model is indicated.

Literature Review

Many works have been done in the direction of production inventory. Goyal and Nebebe [11] determined a production and shipment policy for product supplied to a single buyer by a vendor and tried to find out a lower cost inventory policy. Yang and Wee [34] by considering the aspect of both the producer and the buyer developed a mathematical model subject to multi-lot-size production and distribution for deteriorating items with constant production and demand rate. Sana [28] developed a production inventory model to determine the optimal product reliability and production rate that achieve the biggest total integrated profit for an imperfect manufacturing process.Jaggi and Khanna [13] developed a profit maximising inventory model for deteriorating imperfect quality products under inflation and permissible delay in payment.Giri and Roy[10] considered a single- vendor single buyer production inventory model for a single product and the optimal decisions are obtained by minimising the average total cost of the integrated system. Zhang and Xu [35] investigated multi-item production planning with carbon cap and trade mechanism and proposed a profit maximization model to characterize the optimization problem. Cardenas-Barron and Sana [4] developed a production inventory model for two echelon supply chain consisting of one manufacturer and one retailer considering the procurement cost per unit as function of the production rate. Sivashankari and Panayappan[30] developed production inventory model for deteriorating items and considered two different rates of productions and investigated two production inventory models with shortages and without shortages They derived global optimal solution. Chakrabarty et al.[5] developed a production inventory model of a single product with imperfect production process in which inflation and time value of money considered under shortages.Ghosh et al. [9] developed a production inventory model with selling price discount under random machine break down during production. Jain et al. [14] developed production inventory model by considering repair policy of imperfect items with time dependent demand, production and repair rates under inflationary conditions. Yadav et al. [33] proposed a supply chain model with imperfect quality items considering the demand function depends on the marketing expenditure and the selling price of the buyer.

Palanivel and Gowri [24] developed a production inventory model with delayed deteriorating items in which demand is a deterministic function of selling price. Kumar et al. [17] proposed a mathematical model for a new product launch strategy considering free replacement during warranty period and reworking during production process.Manna et al. [18] proposed a two-plant production inventory model for imperfect product under fuzzy environment. Jayaswal et al. [15] analysed a fiscal construction featured model for imperfect quality items with trade credit policy. Rahman et al. [26] developed a production inventory model where the demand and the inventory cost parameters are interval valued. Das et al. [6] formulated a production inventory model considering product's replacement facility of the failure product within guarantee period to their customers. Mishra et al. [21] developed a sustainable production inventory model where they proposed a solution methodology for determining the optimal strategies of cycle time, green technology investment and the fraction period length in positive inventory level. Pan et al. [25] developed sustainable production inventory to reduce carbon emission in which they considered the buyer and vendor in the integrated supply chain and agree to co-invest funds to reduce carbon emission under carbon cap and trade and carbon tax policies.

The items produced is transported from the source to a given place for consumptions that depend upon the population size of that place and a lot of work has been done in the context of population growth, logistic growth. Wali et al. [31] studied the population growth of Uganda and focused on the application of logistic equation to model the population growth of Uganda and used least square method to find the population growth rate, the carrying capacity and when the population of Uganda will approximately be half of the value of its carrying capacity. Ofroi et al. [23] developed mathematical models to predict the population growth of Ghana and they applied exponential and logistic growth model to predict the population growth of Ghana using data from 1960 to 2011 and concluded that exponential model gave a good forecasting result than logistic model. Wei et al. [32] studied two population growth models and tried to find out a proper way to explain and predict the population growth where they differentiate human population growth and biological population growth. Moreover, they analysed the factors affecting population growth in China. Mwakisisle and Mushi [22] developed a mathematical model for Tanzania population growth by using exponential and logistic population growth model, and predicted the population for the period of 2013 to 2035. Budiono et al. [2] developed a logistic growth model where environmental carrying

Table 1 Related works and their contributions

Authors	Demand type	Production rate depends on population size	Carbon emission	Green investment	Demand represents average consumption of a population
Ritha and Poongodisathiya[27]	Constant	No	Yes	Yes	No
Palanivei and Gowri [24]	Quadratic and price dependent	No	No	No	No
Gautam &Kahnna[8]	Constant	No	Yes	No	No
Shen et al. [29]	Constant	No	Yes	No	No
Mashud et al. [19]	Constant	No	No	No	No
Mishra et al. [20]	Constant	No	Yes	Yes	No
Das et al. [6]	Replacement Period-, stock-, price-dependent	No	No	No	No
Gautam et al. [7]	Price dependent	No	Yes	No	No
Kamna et al. [16]	Price dependent	No	No	No	No
Halim et al. [12]	Price and stock dependent	No	No	No	No
Buttar & Sharma [3]	Time dependent	No	No	No	No
Bhattacharjee & Sen [1]	Price dependent	No	Yes	Yes	No
This Paper	Time dependent	Yes	Yes	Yes	Yes

capacity is a function of time. A comparison of the present work with some of the related research works are presented in Table 1.

Notations

This section provides the notations used throughout this article.

Symbol	Unit	Meaning
$J(t)$	Constant	Population size at time t
q_0	Constant	Initial population
α	Constant	Rate of decrease of population $0 < \alpha < 1$
Δ	Constant	Rate of immigration and migration
μ	Constant	Average consumption per individual

Symbol	Unit	Meaning
$Q(t)$	Ton/time unit	Amount of grain produced at time t
θ	Constant	Rate of deterioration $0 < \theta < 1$
P	Ton /time unit	Production rate
O	\$	Setup cost
c	\$/time unit	Deterioration cost per unit per unit time
c_T	\$/time unit	Transportation cost per unit per unit time
ρ	\$/unit	Fixed production cost
ε	\$/unit/time unit	Production cost per unit per unit time
c_e	\$/unit/time unit	Carbon tax in production per unit production per unit time
c_{he}	\$/unit/time unit	Carbon tax in holding per unit per unit time
h	\$/unit/time unit	Holding cost per unit per unit time

Assumptions

The model is formulate based on the following assumptions:

- Lead time is zero
- $\Delta = immigration - migration$, if migration is greater than immigration then $\Delta < 0$, if migration is less than immigration then $\Delta > 0$.
- Demand of grain is $D(t) = \mu J(t)$
- Production cost $= \rho P + \frac{\varepsilon D(t)}{P}$, $0 \leq t \leq T$
- Planning horizon is infinite
- Production rate is greater than demand.
- Carbon emission is considered in the model.
- The green investment is assumed in one case and the carbon emission cost for that case is defined as $c_e(I) = c_e e^{-aI_1}$, where $0 < a < 1$ is the sensitivity of green investment.
- Preservation is considered in one case and the cost of deterioration is defined as $c(\beta) = c\theta e^{-w\beta}$ where, $0 < w < 1$ is the sensitivity of preservation cost, Gautam et al. [7]

Description of Problem

The demand and supply chain of essential food items transported from the source to a given location depends on the population size and rate of consumption. In literature, the population size follows the logistic, exponential growth, logarithmic growth and exponential decay model which is govern by first order separable equation. In reality the rate of change of population not only depends on the birth and death rate but also on the migration and immigration. Thus the demand for the particular grain depends on the size of population immigrated and migrated in the given location. The inventory process can be pictorially depicted in Fig. 1. Consider an agricultural field with a definite output rate of a single agri-product. The produced item is transported for the consumption by the population in a certain region. The population size changes with time and therefore to maintain the supply change it is necessary to determine the production rate. The population decays exponentially and the rate of change of population

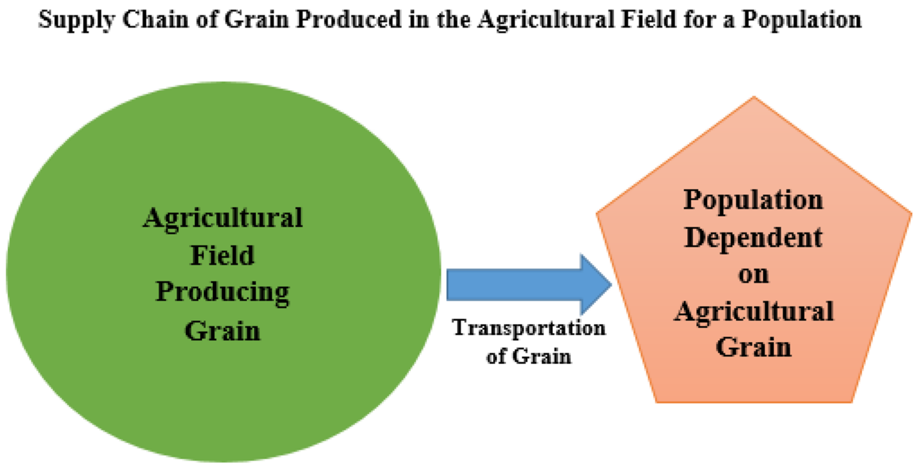

Fig. 1 Pictorial representation of inventory model

size also depends on the immigration and migration of individuals. As a result, producers in that region must supply the consumable agri-product throughout the planning horizon. Therefore, the ideal production rate is a critical decision-making criterion for the producer in order to reduce the cost of production, maintenance, and transportation.The model is analysed under the following objectives;

I. Minimize the overall cost of production and optimize the production rate.
II. Minimize the overall cost of production considering the holding cost in the objective function.
III. Minimize the overall cost of production considering the carbon tax as a function of green investment.
IV. Considering preservation cost in the model the overall cost of production is minimized.

Model Formulation

Population Growth Model

The differential equation governing the growth of population size in a given region in the time period $[0, T]$ is given by;

$$\frac{dJ}{dt} = -\alpha J + \Delta$$

Subject to $J = q_0$ at $t = 0$

The population size of the region at any time t is given by;

$$J(t) = q_0 e^{-\alpha t} + \frac{\Delta}{\alpha}\left(1 - e^{-\alpha t}\right) \tag{1}$$

Let μ be the average consumption rate of individual at time t then total consumption made by the population can be obtained using Eq. 1 as follows;

$$D(t) = \mu J(t) \forall t \in [0, T] \tag{2}$$

Thus, from Eqs. 1 and 2 the demand of the grain in the population at any time t is given by;

$$D(t) = \mu\left[q_0 e^{-\alpha t} + \frac{\Delta}{\alpha}\left(1 - e^{-\alpha t}\right)\right] \tag{3}$$

Production Inventory Model

The differential equation representing the production inventory model in the time period $[0, T]$ is given by;

$$\frac{dQ}{dt} + \theta Q = P - D(t)$$

Subject to $Q = \mu q_0$ at $t = 0$

The total grain production to meet the population at any time t is given by;

$$Q = \frac{P}{\theta}\left(1 - e^{-\theta t}\right) + \mu\left[q_0 e^{-\theta t}\left(1 - \frac{1}{\theta - \alpha}\right) - \frac{\Delta e^{-\theta t}}{\theta(\theta - \alpha)} - \frac{q_0 e^{-\theta t}}{\theta - \alpha} - \frac{\Delta}{\alpha}\left(\frac{1}{\theta} - \frac{e^{-\alpha t}}{\theta - \alpha}\right)\right]$$

The various costs associated with the production and transportation of grain are given by;

- Setup cost:

$$SC = O \tag{4}$$

- Deterioration cost:

$$DC = c\theta\left[\frac{P}{\theta}\left(T + \frac{e^{-\theta T} - 1}{\theta}\right) + \mu q_0\left(\frac{1 - e^{-\theta T}}{\theta}\right)\left(1 - \frac{1}{\theta - \alpha}\right) + \frac{\Delta\left(e^{-\theta T} - 1\right)}{\theta^2(\theta - \alpha)} + \frac{q_0\left(e^{-\theta T} - 1\right)}{\theta(\theta - \alpha)} - \frac{\mu\Delta}{\alpha}\left(\frac{T}{\theta} - \frac{\left(e^{-\theta T} - 1\right)}{\theta(\theta - \alpha)}\right)\right] \tag{5}$$

- Transportation cost:

$$TC = c_T\mu\left[\left(\frac{q_0}{\alpha} + \frac{\Delta}{\alpha^2}\right)\left(1 - e^{-\alpha T}\right) + \frac{\Delta T}{\alpha}\right] \tag{6}$$

- Carbon emission cost:

$$CEC = c_e P T \tag{7}$$

- Production cost:

$$PC = \rho PT + \varepsilon\mu\left[\left(\frac{q_0}{\alpha} + \frac{\Delta}{\alpha^2}\right)\left(1 - e^{-\alpha T}\right) + \frac{\Delta T}{\alpha}\right] \tag{8}$$

The average total cost of production of grain for the population in the time interval $[0, T]$ can be obtained by adding all the expressions from 4–8

$$\begin{aligned} ATC = \frac{1}{T}\Bigg[& O + c\theta\left[\frac{P}{\theta}\left(T + \frac{e^{-\theta T} - 1}{\theta}\right)\right. \\ & + \mu\left[q_0\left(\frac{1 - e^{-\theta T}}{\theta}\right)\left(1 - \frac{1}{\theta - \alpha}\right) + \frac{\Delta\left(e^{-\theta T} - 1\right)}{\theta^2(\theta - \alpha)} + \frac{q_0\left(e^{-\theta T} - 1\right)}{\theta(\theta - \alpha)} - \frac{\Delta}{\alpha}\left(\frac{T}{\theta} - \frac{\left(e^{-\theta T} - 1\right)}{\theta(\theta - \alpha)}\right)\right]\Bigg] \\ & + c_T\mu\left[\left(\frac{q_0}{\alpha} + \frac{\Delta}{\alpha^2}\right)\left(1 - e^{-\alpha T}\right) + \frac{\Delta T}{\alpha}\right] + c_e PT + \rho P + \frac{\varepsilon\mu}{P}\left[\left(\frac{q_0}{\alpha} + \frac{\Delta}{\alpha^2}\right)\left(1 - e^{-\alpha T}\right) + \frac{\Delta T}{\alpha}\right]\Bigg] \end{aligned} \tag{9}$$

The decision variables for the formulated model are: (i) Production rate P, and (ii) Time cycle T

Solution Procedure

Due to extremely nonlinear nature of the cost function, it is difficult to establish the sufficient condition of optimality through analytical method. Therefore, the solution procedure of Gautam et al. (2020) is implemented to obtain the optimal solution graphically for cases with two decision variables and inbuilt optimization tool of MATHEMATICA for the cases with more than two variables. Thus, in order to proceed further a solution procedure is provided to discuss the steps to obtain the solution.

Solution Procedure

Step 1 Set the values of different system parameters
Step 2 Use the optimization tool to obtain the optimal values of decision variable P^* and T^*
Step 3 Check the decision variable satisfy the condition $P^* > 0$
Step 4 Obtain the minimum value of the average total cost
Step 5 Display the solution surface graphically if number of decision variable is less than 3.

Numerical Illustration

The present inventory model is based on the economic production quantity with respect to the average consumption rate of a population changing with time. For the computational purpose the agri-producer considered in the model is a farmer and the agri-product is grain. So, there is no such EPQ model reported in the past with similar idea of demand function therefore the values of all the parameters cannot be available from the single source to validate the model.Thus, in the absence of authenticated data we have consider a hypothetical system with values of different parameters involved in the model.

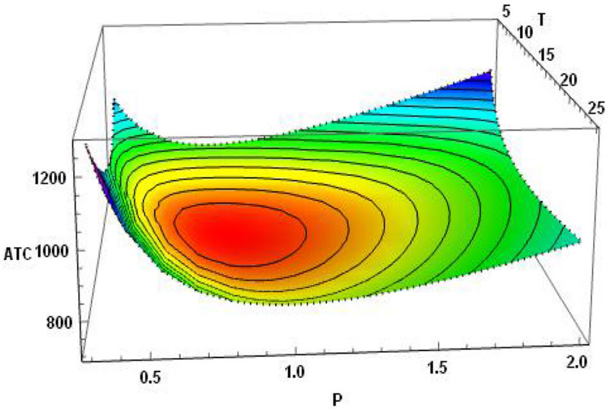

Fig. 2 Graphical representation of convexity of cost Function w.r.t. example 8.1

Example 8.1 Anagro producer distributing the grain to a region having initial population size $q_0 = 100000$, which is decreasing at a rate of $\alpha = -0.05$. The average consumption rate of grain is $\mu = 0.0001$. The produced grain deteriorates at a rate $\theta = 0.01$. The population changes due to migration and immigration is $\Delta = 100$. The different costs involved in the system are as below;

Deterioration cost $c = \$0.8$, Transportation cost $c_T = \$10$, Setup cost $O = \$1000$,

Fix production cost $\rho = \$100$, Unit production cost $\varepsilon = \$15$, Carbon tax $c_e = \$200$

Solution: The average total cost of production to maintain the demand of grain of a population is $ATC = \$697.402$ for the period $T = 10.0061$ month with the production rate $P = 0.791056$ ton/unit time. The convexity is shown in Fig. 2

Special Cases

Case 9.1

When holding cost is considered.

If the produced grains are stored before they are supplied to the destination. Then two different costs are included in the average total cost; *holding cost and carbon emission cost in holding*.Thus the cost function can be obtained by adding holding cost to the cost Eq. 9;

$$AATC = \frac{1}{T}\left[\begin{array}{l} O + c\theta\left[\frac{P}{\theta}\left(T + \frac{e^{-\theta T}-1}{\theta}\right) + \mu\left[q_0\left(\frac{1-e^{-\theta T}}{\theta}\right)\left(1 - \frac{1}{\theta-\alpha}\right) + \frac{\Delta\left(e^{-\theta T}-1\right)}{\theta^2(\theta-\alpha)}\right.\right. \\ \left.\left. + \frac{q_0\left(e^{-\theta T}-1\right)}{\theta(\theta-\alpha)} - \frac{\Delta}{\alpha}\left(\frac{T}{\theta} - \frac{\left(e^{-\theta T}-1\right)}{\theta(\theta-\alpha)}\right)\right]\right] \\ + \left(h + c_{he}\right)\left[\begin{array}{l} \frac{P}{\theta}\left(T + \frac{e^{-\theta T}-1}{\theta}\right) \\ + \mu\left[q_0\left(\frac{1-e^{-\theta T}}{\theta}\right)\left(1 - \frac{1}{\theta-\alpha}\right) + \frac{\Delta\left(e^{-\theta T}-1\right)}{\theta^2(\theta-\alpha)} + \frac{q_0\left(e^{-\theta T}-1\right)}{\theta(\theta-\alpha)} - \frac{\Delta}{\alpha}\left(\frac{T}{\theta} - \frac{\left(e^{-\theta T}-1\right)}{\theta(\theta-\alpha)}\right)\right] \end{array}\right] \\ + c_T\mu\left[\left(\frac{q_0}{\alpha} + \frac{\Delta}{\alpha^2}\right)\left(1 - e^{-\alpha T}\right) + \frac{\Delta T}{\alpha}\right] + c_e PT + \rho PT + \frac{\varepsilon\mu}{P}\left[\left(\frac{q_0}{\alpha} + \frac{\Delta}{\alpha^2}\right)\left(1 - e^{-\alpha T}\right) + \frac{\Delta T}{\alpha}\right] \end{array}\right] \tag{10}$$

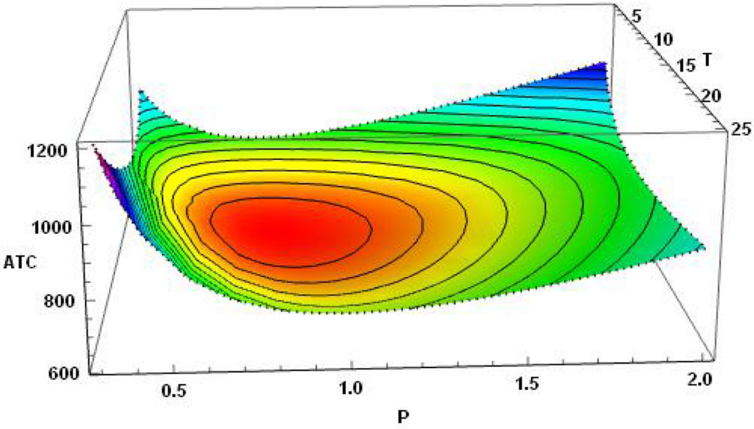

Fig. 3 Graphical Representation of convexity of cost Function w.r.t. example 9.1

The decision variables for the formulated model are (i) Production rate P, and (ii) Time cycle T

Example 9.1 Anagro producer distributing the grain to a region having initial population size $q_0 = 100000$, which is decreasing at a rate of $\alpha = -0.05$. The average consumption rate of grain is $\mu = 0.0001$. The produced grain deteriorates at a rate $\theta = 0.01$. The population changes due to migration and immigration is $\Delta = 100$. The different costs involved are as below;

Holding cost $h = \$0.1$, Carbon tax in holding $c_{he} = \$0.2$, Deterioration cost $c = \$0.8$
Transportation cost $c_T = \$10$, Setup cost $O = \$1000$, Fix production cost $\rho = \$100$
Unit production cost $\varepsilon = \$15$, Carbon tax $c_e = \$200$

Solution: The average total cost of production to maintain the demand of grain of a population is $ATC = \$608.402$ for the period $T = 9.74899$ month with the production rate $P = 0.786415$ ton/unit time. The convexity is shown in Fig. 3.

Case 9.2

When Green Investment is considered.

If the producer invests in green policies to reduce the carbon footprints in all the three possible sectors that is; holding, production and transportation then three more variables are included in the model. Thus, the cost equation can be obtained by adding the green investment in the Eq. 9

$$ATC = \frac{1}{T}\left[\begin{array}{l} O + c\theta\left[\frac{P}{\theta}\left(T + \frac{e^{-\theta T}-1}{\theta}\right) + \mu\left[\begin{array}{l} q_0\left(\frac{1-e^{-\theta T}}{\theta}\right)\left(1 - \frac{1}{\theta-\alpha}\right) + \frac{\Delta\left(e^{-\theta T}-1\right)}{\theta^2\left(\theta-\alpha\right)} + \frac{q_0\left(e^{-\theta T}-1\right)}{\theta\left(\theta-\alpha\right)} \\ -\frac{\Delta}{\alpha}\left(\frac{T}{\theta} - \frac{\left(e^{-\theta T}-1\right)}{\theta\left(\theta-\alpha\right)}\right) \end{array}\right]\right] \\ +c_T\mu\left[\left(\frac{q_0}{\alpha} + \frac{\Delta}{\alpha^2}\right)\left(1 - e^{-\alpha T}\right) + \frac{\Delta T}{\alpha}\right] + \frac{c_e e^{-aI_1} PT}{\theta} + \rho PT + \frac{\varepsilon\mu}{P}\left[\left(\frac{q_0}{\alpha} + \frac{\Delta}{\alpha^2}\right)\left(1 - e^{-\alpha T}\right) + \frac{\Delta T}{\alpha}\right] + I_1 \end{array}\right] \tag{11}$$

where I_1 is the total green investment, $0 < a < 1$ sensitivity of green investment and the decision variables in this case are (i) Production rate P (ii) Green Investment I_1, and (iii) Time cycle T

Example 9.2 A agro producer distributing the grain to a region having initial population size $q_0 = 100000$, which is decreasing at a rate of $\alpha = -0.05$. The average consumption rate of grain is $\mu = 0.0001$. The produced grain deteriorates at a rate $\theta = 0.01$. The population changes due to migration and immigration is $\Delta = 100$. The different costs involved are as below;

Deterioration cost $c = \$0.8$, Transportation cost $c_T = \$10$, Setup cost $O = \$1000$

Fix production cost $\rho = \$100$, Unit production cost $\varepsilon = \$15$,Carbon tax $c_e = \$200$

Sensitivity of green investment $a = 0.1$

Solution: The average total cost of production to maintain the demand of grain of a population is $ATC = \$584.032$ for the period $T = 11.4256$ month with the production rate $P = 1.348$ tom/unit time under the green investment $I_1 = \$78.9953$

Case 9.3

When preservation is considered.

If preservation is allowed, then the rate of deterioration further reduces and thus the average total cost can be obtained by adding preservation cost to the cost Eq. 9

$$ATC = \frac{1}{T}\left[\begin{array}{l} O + c\theta e^{-w\beta}\left[\begin{array}{l}\frac{P}{\theta}\left(T + \frac{e^{-\theta T}-1}{\theta}\right) + \mu\left[\begin{array}{l} q_0\left(\frac{1-e^{-\theta T}}{\theta}\right)\left(1-\frac{1}{\theta-\alpha}\right) + \frac{\Delta\left(e^{-\theta T}-1\right)}{\theta^2(\theta-\alpha)} + \frac{q_0\left(e^{-\theta T}-1\right)}{\theta(\theta-\alpha)} \\ -\frac{\Delta}{\alpha}\left(\frac{T}{\theta} - \frac{\left(e^{-\theta T}-1\right)}{\theta(\theta-\alpha)}\right)\end{array}\right]\end{array}\right] \\ +c_T\mu\left[\left(\frac{q_0}{\alpha}+\frac{\Delta}{\alpha^2}\right)\left(1-e^{-\alpha T}\right)+\frac{\Delta T}{\alpha}\right] + c_e PT + \rho P + \frac{\varepsilon\mu}{P}\left[\left(\frac{q_0}{\alpha}+\frac{\Delta}{\alpha^2}\right)\left(1-e^{-\alpha T}\right)+\frac{\Delta T}{\alpha}\right] + \beta \end{array}\right] \tag{12}$$

Where β is effective preservation and $0 < w < 1$ is cost sensitivity of preservation The decision variables in this case are (i) Production rate P (ii) Preservation cost β and (iii) Time cycle T

Example 9.3 Aagro producer distributing the grain to a region having initial population size $q_0 = 100000$, which is decreasing at a rate of $\alpha = -0.05$. The average consumption rate of grain is $\mu = 0.0001$. The produced grain deteriorates at a rate $\theta = 0.01$. The population changes due to migration and immigration is $\Delta = 100$. The different costs involved are as below.

Deterioration cost $c = \$0.8$, Transportation cost $c_T = \$10$, Setup cost $O = \$1000$
Fix production cost $\rho = \$100$, Unit production cost $\varepsilon = \$15$, Sensitivity of preservation cost $w = 0.01$

Solution: The average total cost of production to maintain the demand of grain of a population is $ATC = \$697.402$ for the period $T = 9.74899$ month with the production rate $P = 0.786415$ ton/unit time and preservation cost $\beta = \$0$.

Sensitivity Analysis

In this section sensitivity analysis of the parameters is performed in order to test the flexibility of the model. The change in the parameters is defined by $N \rightarrow N * (1 + r)$ where $r = \{-0.4, -0.2, 0.2, 0.4\}$. The analysis of the parameters is performed with respect to the optimal solution obtained in Problem 8.1. and presented in Table 2.

Based on the sensitivity analysis, the impact of the changes of different system parameters on the optimal solution are presented in Table 3

Managerial Implications

In accordance with the numerical results and sensitivity analyses provided in Tables 2,3 we provide the following managerial insights;

- Average total cost and production rate decreases with the decrease in setup cost and the change is quite noticeable. However, the decrease in planning horizon is significant with the decrease in setup cost.
- The change in average total cost is mild when we made a change in fixed production cost.
- Rate of deterioration, unit cost of deterioration and the rate of immigration and migration have almost no effect on average total profit. So, these parameters are insensitive in the model.
- The average total cost and rate of production increases significantly when the initial population size increases but the change is almost insignificant if the initial size of the population decreases.
- Average total cost is affected with the change in consumption factor. The average total cost and rate of production increases with the increase in average consumption rate. However, the planning horizon decreases with the increase in consumption rate.
- Average total cost and rate of production increase with the increase in unit production cost but planning horizon decreases with the increase in unit production cost.
- A noticeable increase in average total cost is observed with the increase in carbon emission cost and transportation cost.
- Comparing the cases, it can be concluded that incorporating the green investment is a potential decision to increase the price of the product. Further the preservation cost has no impact on the average total cost.

Conclusion

This deals with a single item production inventory model to study the production rate and planning horizon of a production sector producing grain for a time reliant population size. The demand of the production inventory is the average consumption of the grain by the population. The holding, transportation and deterioration cost are all constant. The effect of carbon emission is considered in the model along with the decarbonisation through green investments. The change in population size is represented by the population decay model together with the impact of immigration and migration factor.The demand of the product changes with the change in the size of population with time. Precisely, the demand decreases with the increase in time since population follows decay model. Holding cost and green investment is not considered in example 8.1 in order to keep the system realistic and in

Table 2 Sensitivity analysis

Parameters	% Change	Optimal values			% change in optimal values		
		T (month)	P (Ton/unit time)	ATC ($)	T (month)	P (Ton/unit time)	ATC ($)
$O = 1000$	− 40%	7.95604	0.769232	652.928	− 20.4881	− 2.7588	− 6.3770
	− 20%	9.05876	0.780856	676.427	− 9.4676	− 1.2894	− 3.0075
	20%	10.8438	0.800248	716.583	8.3719	1.1620	2.7503
	40%	11.5993	0.808676	734.403	15.9223	2.2274	5.3055
$\rho = 100$	− 40%	10.2124	0.852138	664.582	2.0617	7.7215	− 4.7060
	− 20%	10.1058	0.819941	681.297	0.9963	3.6514	− 2.3092
	20%	9.91239	0.764955	712.958	− 0.9365	− 3.2995	2.2305
	40%	9.82411	0.741218	728.016	− 1.8187	− 6.3002	4.3897
$\theta = 0.01$	− 40%	10.0099	0.791119	698.214	0.0379	0.0079	0.1164
	− 20%	10.0081	0.791089	697.788	0.0199	0.0041	0.0558
	20%	10.0037	0.791022	697.051	− 0.0239	− 0.0042	− 0.0503
	40%	10.0012	0.790985	696.731	− 0.0489	− 0.0089	− 0.0962
$\alpha = -0.05$	− 40%	8.27852	0.812737	738.801	− 17.2652	2.7407	5.9361
	− 20%	9.03244	0.802623	719.131	− 9.7306	1.4622	3.1157
	20%	11.3302	0.777269	672.672	13.2329	− 1.7428	− 3.5460
	40%	13.279	0.759610	643.042	32.7090	− 3.9751	− 7.7946
$q_0 = 100000$	− 40%	9.99125	0.796683	702.689	− 0.1484	0.7113	0.7580
	− 20%	9.99866	0.793875	700.049	− 0.0743	0.3563	0.3795

Table 2 (continued)

Parameters	% Change	Optimal values			% change in optimal values		
		T (month)	P (Ton/unit time)	ATC ($)	T (month)	P (Ton/unit time)	ATC ($)
	20%	9.4654	0.862799	769.39	− 5.4037	9.0692	10.3223
	40%	9.02403	0.928372	836.894	− 9.8147	17.3585	20.0016
$\Delta = 100$	− 40%	9.99125	0.796683	702.689	− 0.1484	0.7113	0.7580
	− 20%	9.99866	0.793875	700.049	− 0.0743	0.3563	0.3795
	20%	10.0134	0.788227	694.749	0.0729	− 0.3576	− 0.3804
	40%	10.0208	0.785387	692.089	0.1469	− 0.7166	− 0.7618
$c = 0.8$	− 40%	10.0089	0.791108	698.351	0.0279	0.0065	0.1360
	− 20%	10.0075	0.791082	697.876	0.0139	0.0032	0.0679
	20%	10.0046	0.791031	696.927	− 0.0149	− 0.0031	− 0.0681
	40%	10.0032	0.791005	696.453	− 0.0289	− 0.0064	− 0.1360
$C_T = 10$	− 40%	10.688	0.798526	646.878	6.8148	0.9443	− 7.2446
	− 20%	10.3283	0.794573	672.258	3.2200	0.4445	− 3.6053
	20%	9.71494	0.7879	722.334	− 2.9098	− 0.3989	3.5749
	40%	9.45004	0.785045	747.078	− 5.5572	− 0.7598	7.1230
$c_e = 200$	− 40%	10.4505	0.929403	629.017	4.4412	17.4889	− 9.8056
	− 20%	10.2124	0.852138	664.582	2.0617	7.7215	− 4.7060
	20%	9.82411	0.741218	728.016	− 1.8187	− 6.3001	4.3897
	40%	9.66149	0.699564	756.808	− 3.4439	− 11.5658	8.5181

Table 2 (continued)

Parameters	% Change	Optimal values			% Change in optimal values		
		T (month)	P (Ton/unit time)	ATC ($)	T (month)	P (Ton/unit time)	ATC ($)
$\varepsilon = 15$	− 40%	10.7328	0.618919	589.885	7.2625	− 21.7604	− 15.4167
	− 20%	10.3273	0.710677	647.178	3.2100	− 10.1609	− 7.2015
	20%	9.74038	0.863403	742.625	−2.6555	9.1456	6.4844
	40%	9.51413	0.929693	784.073	− 4.9167	17.5255	12.4276
$\mu = 0.0001$	− 40%	11.6061	0.626457	539.163	15.9902	− 20.8075	− 22.6897
	− 20%	10.6864	0.714207	622.277	6.7988	− 9.7147	− 10.7721
	20%	9.47145	0.860225	766.903	− 5.3432	8.7438	9.9657
	40%	9.03425	0.923612	832.172	− 9.7125	16.7568	19.3245

Table 3 Impact on the decision variables on increasing the inventory parameters

Parameters	T (month)	P (Ton/unit time)	ATC ($)
Setup cost O	↑	↑	↑
Fixed production cost ρ	↓	↓	↑
Rate of deterioration θ	↓	↓	↓
Rate of decrease of population α	↑	↓	↓
Initial Population q_0	↓	↑	↑
Rate of immigration and migration Δ	↑	↓	↓
Deterioration cost per unit per unit time c	↓	↓	↓
Transportation cost per unit per unit time C_T	↓	↓	↑
Carbon tax in production per unit production per unit time c_e	↓	↓	↑
Production cost per unit per unit time ε	↓	↑	↑
Average consumption per individual μ	↓	↑	↑

that way it is possible to discuss the impact of holding the goods in example 9.1, making investments on green technologies in example 9.2 and considering preservation in example 9.3, in the rate of production and overall cost. Form the sensitivity analysis it is observed that the average consumption affects the average total cost of production and the rate of production which is realistic in nature. Further, the increase in the initial population will magnify the cost and rate of production.It is observed that all the parameters are sensitive in nature, though the influence of a parameter on the decision variables and overall production cost may varies.The proposed model is investigated under three different cases and a remarkable observation that can be made from the results is that, the average total cost is least when green investment is implemented to reduce the carbon emission cost.The optimal is obtained by using MATHEMATICA.

The present work is an extension of the current literature on production inventory and no research work on production inventory with the demand function based on consumption of a dynamic population is not reported in the literature of production inventory as far as the author's knowledge is concerned. The proposed model can be extended to study the optimal production rate and planning horizon for the consumption of dynamic population following logistic growth.

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Declarations

Conflict of interest The authors don't have any competing interest.

Human and animal rights No animal is used while conducting the research.

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